ANALYZING FUZZY RELIABILITY OF SERIES-PARALLEL SYSTEM

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Abstract

The reliability of a system is the probability that the system will perform a specified function satisfactorily during some interval of time under specified operating conditions. In general fuzzy sets are used to analyze the system reliability. In this paper we present a new method for analyzing fuzzy reliability of series-parallel network systems based on vague set theory, where the reliabilities of components of a system are represented by vague sets defined in the universe of discourse [0, 1].

Keywords: - fuzzy system reliability, series - parallel network system, vague set.

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Introduction

Fuzzy set theory proposed by Zadeh (1965) [1] permits the replacement of the sharp boundaries in classical set theory by fuzzy boundaries. The concept of belongingness of an element in the context of classical sets changes to membership grade of the element to certain degree in fuzzy sets. The membership grade of an element *x* of the fuzzy set is given by a real number between zero and one. Due to fuzzy boundaries, this single value for the membership grade is the result of the combined effect of evidences in favour and against the inclusion of the element in the set the utility of the application of fuzzy sets depends on the capability of the user to construct appropriate membership functions, which are often very precise. In many contexts it is difficult to assign a particular real number as a membership grade and in such cases it may be useful to identify meaningful lower and upper bounds for the membership grade. Such generalization of fuzzy sets is called vague sets.

Most of the researches [2 4, 5, 6, 8,9,10] in classical reliability theory are based on binary state assumption for states. In gracefully degradable systems, it is unrealistic to assume that the system possesses only two states that is, 'working' or 'failed'. Such systems may be considered working to certain degrees at different states of its degradation during its transition from fully working state to completely failed state. The degree may be any real number between 0 and 1. Degree 0 would represent the system in completely failed state while a fully working state would be represented by degree1. The assignment of the degree may depend upon the limit of tolerance of the user about the adequate performance of the system. Zadeh [1] suggested a paradigm shift from the theory of total denial & affirmation to a theory of grading, to give new concept of sets called fuzzy sets. Fuzzy sets can express the gradual transition of the system from working state to failed state. The crisp set theory only dichotomizes the system in working state and failed state but fuzzy state theory can cover up all possible states between a fully working state and completely failed state. This approach to the reliability theory is known as Profust reliability, wherein the binary state assumption is replaced by fuzzy state assumption.

Concept of vague sets given by Gau and Buehrer takes into account the favourable and unfavourable evidences separately providing a lower and an upper bound within which the membership grade may lie. Chen[8] presented similarity measures between vague sets. Recently, Chen proposed fuzzy system reliability analysis based on vague set theory, where the reliabilities of the components of a system are represented by vague sets defined in the universe of discourse

[0 1]. Chen's method has the advantages of modeling and analyzing the fuzzy system reliability in a more flexible and more intelligent manner. However, Chen's method limits its applicability to some special case of general vague set.

2. Basic Concepts of Vague Sets

Vague set: - A vague set $\tilde{A}$ in the universe of discourse X is characterized by a membership function $\mu_{\tilde{A}}: X \rightarrow [0,1]$ and a non-membership function $\nu_{\tilde{A}}: X \rightarrow [0,1]$. The grade of membership for any element x in the vague set is bounded by a sub interval [$\mu_{\tilde{A}}(x)$, 1 - $\nu_{\tilde{A}}(x)$], where the grade $\mu_{\tilde{A}}(x)$ is called lower bound of membership grade of x derived from evidences for x and $\nu_{\tilde{A}}(x)$ is the lower bound of membership grade on the negation of x derived from the evidences against x and $\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1$. In the extreme case of equality $\mu_{\tilde{A}}(x) = 1 - \nu_{\tilde{A}}(x)$, the vague set reduces to the fuzzy set with interval value of the membership grade reducing to a single value $\mu_{\tilde{A}}(x)$. In general, however,

$$\mu_{\tilde{A}}(x) \leq \text{exact membership grade of } x \leq 1 - \nu_{\tilde{A}}(x).$$

Expressions (1) and (2) given below can be used to represent a vague set $\tilde{A}$ for finite and infinite universe of discourse X respectively.

$$\tilde{A} = \sum_{k=1}^{n} [\mu_{\tilde{A}}(x_k), 1 - \nu_{\tilde{A}}(x_k)] / x_k$$

$$\tilde{A} = \int_X [\mu_{\tilde{A}}(x_k), 1 - \nu_{\tilde{A}}(x_k)] / x_k$$

A vague set is represented pictorially as

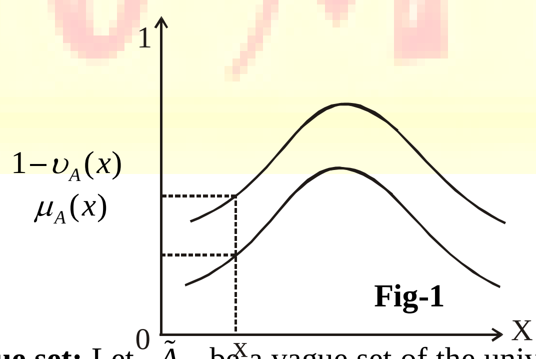


Fig-1

2.1 Convex vague set: Let $\tilde{A}$ be a vague set of the universe of discourse X with $\mu_{\tilde{A}}$ and $\nu_{\tilde{A}}$ as its membership and non-membership functions respectively. The vague set is convex if and only if for every x_1, x_2 in X

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq Min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$$

$$1-\nu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq Min(1-\nu_{\tilde{A}}(x_1), 1-\nu_{\tilde{A}}(x_2)),$$

where $\lambda \in [0,1]$.

2.2 Normal vague set: A vague set $\tilde{A}$ in the universe of discourse X is called normal if $\exists x_i \in X$, such that $1-\nu_{\tilde{A}}(x_i)=1$. That is $\nu_{\tilde{A}}(x_i)=0$.

2.3 Vague number: A vague number is a vague subset in the universe of the discourse X which is both convex and normal.

2.4. Triangular vague number: Chen defined triangular vague sets and arithmetic operations between them. On similar lines we introduce concept of a triangular vague number.

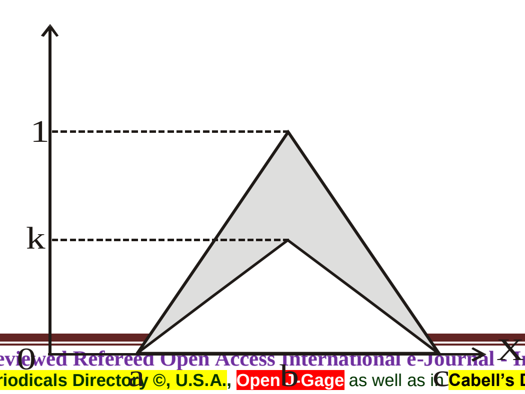
A triangular vague number $\tilde{A}$ denoted by [(*a*,*b*,*c*); *k* ;1] is characterized by a pair of membership functions: a lower membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{k(x-a)}{b-a}, a \leq x \leq b \\ \dfrac{k(c-x)}{c-b}, b \leq x \leq c \\ 0, otherwise \end{cases}$$

and an upper membership function

$$\mu'_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}, a \leq x \leq b \\ \dfrac{c-x}{c-b}, b \leq x \leq c \\ 0, otherwise \end{cases}$$

where $\mu_{\tilde{A}}'(x) = 1-\nu_{\tilde{A}}(x)$ and $k \in [0,1]$. Figure 2 shows a triangular vague number.



When k = 1, triangular vague number reduces to a triangular fuzzy number. In what follows now onwards, we shall use vague number for a triangular vague number.

2.5 Vague point: In a triangular vague number $\tilde{A} = [(a,b,c);k;1]$, if $a=c=b$, say, then

$$\tilde{A} = [(b,b,b);k;1] = b_k ,$$

is said to be a vague point. A vague point b_k reduces to a fuzzy point b_1 for k = 1.

2.6 Arithmetic operations of triangular vague sets:

A simple triangular vague set is represented as: $\langle [(a,b,c);\mu_1],[(a,b,c);\mu_2] \rangle$ or more concisely way as $\langle [(a,b,c);\mu_1;\mu_2] \rangle$, as shown in figure 2. From the definition of triangle vague set, we propose four arithmetic operations for triangular vague sets in the following:

Let A and B are two vague sets as shown in figure

If two vague sets $t_A \neq t_B$, *and* $1-f_A \neq 1-f_{B,}$ then the arithmetic operations are defined as:

(2.6.1)

$$A = \langle [(a_1^{'},b_1,c_1^{'});\mu_1],[(a_1,b_1,c_1);\mu_2] \rangle$$

(2.6.2)

$$B = \langle [(a_2^{'},b_2,c_2^{'});\mu_3],[(a_2,b_2,c_2);\mu_4] \rangle$$

(2.6.3)

$$A(+)B = \langle [(a_1^{'},b_1,c_1^{'});\mu_1],[(a_1,b_1,c_1);\mu_2] \rangle + \langle [(a_2^{'},b_2,c_2^{'});\mu_3],[(a_2,b_2,c_2);\mu_4] \rangle$$

$$= \left\langle \begin{array}{l} [(a_1^{'}+a_2^{'},b_1+b_2,c_1^{'}+c_2^{'});\min(\mu_1,\mu_3)], \\ [(a_1+a_2,b_1+b_2,c_1+c_2);\min(\mu_2,\mu_4)] \end{array} \right\rangle$$

(2.6.4)

$$A(-)B = \langle [(a_1^{'}, b_1, c_1^{'}); \mu_1], [(a_1, b_1, c_1); \mu_2] \rangle$$
$$- \langle [(a_2^{'}, b_2, c_2^{'}); \mu_3], [(a_2, b_2, c_2); \mu_4] \rangle$$
$$= \left\langle \begin{array}{l} [(a_1^{'} - c_2^{'}, b_1 - b_2, c_1^{'} - a_2^{'}); \min(\mu_1, \mu_3)], \\ [(a_1 - c_2, b_1 - b_2, c_1 - a_2); \min(\mu_2, \mu_4)] \end{array} \right\rangle$$

(2.6.5)

$$A(\times)B = \langle [(a_1^{'}, b_1, c_1^{'}); \mu_1], [(a_1, b_1, c_1); \mu_2] \rangle$$
$$+ \langle [(a_2^{'}, b_2, c_2^{'}); \mu_3], [(a_2, b_2, c_2); \mu_4] \rangle$$
$$= \left\langle \begin{array}{l} [(a_1^{'} a_2^{'}, b_1 b_2, c_1^{'} c_2^{'}); \min(\mu_1, \mu_3)], \\ [(a_1 a_2, b_1 b_2, c_1 c_2); \min(\mu_2, \mu_4)] \end{array} \right\rangle$$

(2.6.6)

$$A(/)B = \langle [(a_1^{'}, b_1, c_1^{'}); \mu_1], [(a_1, b_1, c_1); \mu_2] \rangle$$
$$+ \langle [(a_2^{'}, b_2, c_2^{'}); \mu_3], [(a_2, b_2, c_2); \mu_4] \rangle$$
$$= \left\langle \begin{array}{l} [(a_1^{'} / c_2^{'}, b_1 / b_2, c_1^{'} / a_2^{'}); \min(\mu_1, \mu_3)], \\ [(a_1 / c_2, b_1 / b_2, c_1 / a_2); \min(\mu_2, \mu_4)] \end{array} \right\rangle$$

3. Fuzzy reliability of series - parallel network system based on vague set

Here we present a new method for analyzing fuzzy system reliability based on vague set theory, where the reliabilities of components of a system are represented by vague sets defined in the universe of discourse [0, 1].

Consider a series-parallel network consisting of 'n' connections connected in parallel and each connection contains 'm' components as shown in the figure -1. The fuzzy reliability is given by $\tilde{R}_{sp} = {}_{k=i}^{n}\otimes(1 \ominus {}_{i=1}^{m}(1-\tilde{R}_{ik}))$ of the series-parallel network shown in figure 4.7. Reliability can be evaluated by the proposed algorithm, where $\tilde{R}_{ik}$ represents the reliability of the kth component at the ith stage.

$$\widetilde{R_{sp}} = {}_{k=1}^{n}\otimes \{1 \ominus [(1 \ominus (r_{1k}, r_{2k} r_{3k}, r_{4k}; r'_{1k}, r'_{2k}, r'_{3k}, r'_{4k})) \otimes \ldots\ldots\ldots\ldots \otimes (1 \ominus (r_{mk}, r_{mk}, r_{mk}, r_{m4}; r'_{mk}, r'_{mk}, r'_{mk}, r'_{mk}))]$$

$$= {}_{k=1}^{n}\otimes [1 - \prod_{j=1}^{n}(1 - r_{jk}), 1 - \prod_{j=1}^{n}(1 - r_{jk}), 1 - \prod_{j=1}^{n}(1 - r_{jk}), 1 - \prod_{j=1}^{n}(1 - r_{jk}); 1 - \prod_{j=1}^{n}(1 - r'_{jk}), 1 - \prod_{j=1}^{n}(1 - r'_{jk}), 1 - \prod_{j=1}^{n}(1 - r'_{jk}), 1 - \prod_{j=1}^{n}(1 - r'_{jk})]$$

$$= [\{\prod_{k=1}^{n}(1 - \prod_{j=1}^{n}(1 - r_{jk})), \prod_{k=1}^{n}(1 - \prod_{j=1}^{n}(1 - r_{jk})), \prod_{k=1}^{n}(1 - \prod_{j=1}^{n}(1 - r_{jk})), \prod_{k=1}^{n}(1 - \prod_{j=1}^{n}(1 - r_{jk}))\}; \{\prod_{k=1}^{n}(1 - \prod_{j=1}^{n}(1 - r'_{jk})), \prod_{k=1}^{n}(1 - \prod_{j=1}^{n}(1 - r'_{jk})), \prod_{k=1}^{n}(1 - \prod_{j=1}^{n}(1 - r'_{jk})), \prod_{k=1}^{n}(1 - \prod_{j=1}^{n}(1 - r'_{jk}))\}]$$

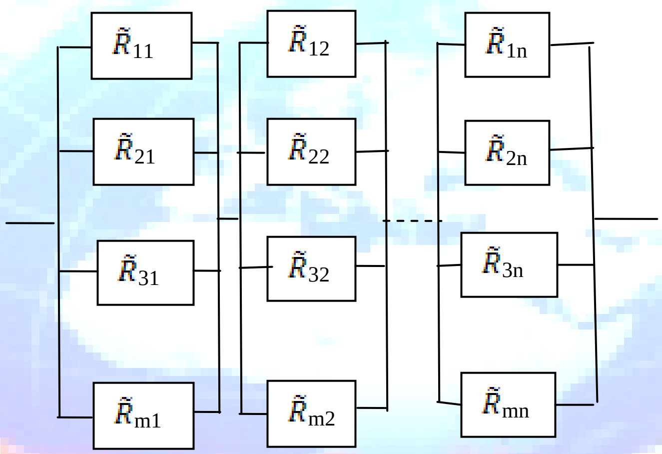


Fig. 1 Series-Parallel networks

6. Conclusion

In this paper we are presented a new method for analyzing fuzzy system reliability of series - parallel network systems using vague set theory, where the component of a system are represented by vague sets defined in the universe of discourse [0, 1]. The proposed method model can analyze the fuzzy system reliability in a more flexible and more intelligent manner. It can provide us with a more flexible and more intelligent way for fuzzy system reliability analysis.

References:

1. L.A. Zadeh, Fuzzy sets. Information Control 8 (1965) 338-353.
2. Lee J.H., Lee-Kwang H., Lee K.M, "A method for ranking fuzzily fuzzy numbers," 2000 IEEE International Fuzzy Systems Conference, vol.1, pp.71-76, SanAntonio,Texas, May. 2000.
3. Lee Young-Hyun, Koop R., "Application of fuzzy control for a hydraulic forging machine. Fuzzy Sets and Systems 118 (2001) 99-108.
4. Lee-Kwang H., Lee J.H., "A method for ranking fuzzy numbers based on a viewpoint and its application to decision making," IEEE Trans. on Fuzzy Systems, vol.7, pp.677-685, Dec. 1999.
5. S.M. Chen, "Measures of similarity between vague sets." Fuzzy Sets & Systems 74(1995) 217-223.
6. Sgarro Andrea, "Possibilistic information theory: a coding theoretic approach" Fuzzy Sets and Systems 132 (2002) 11 – 32.
7. Chen Jen-Yang, "Rule regulation of fuzzy sliding mode controller design: direct adaptive approach, Fuzzy Sets and Systems 120(2001) 159-168.
8. Chen Shyi- Ming, "Analyzing Fuzzy System Reliability Using Vague Set Theory", International journal of Applied Science and Engineering 2003. 1,1: 82-88.
9. Chena Y.H., Wangb Wen-June, Chih-Hui Chiub "New estimation method for the membership values in fuzzy sets", Fuzzy Sets and Systems 112 (2000) 21{525}
10. W.L. Gau, D.J Buchrer., Vague Sets. IEEE Transactions on Systems, Man and Cybernetics, 23 (1993) pp. 610-614.